< 5.5. The Lifting-Surface Theory and VLM Method >

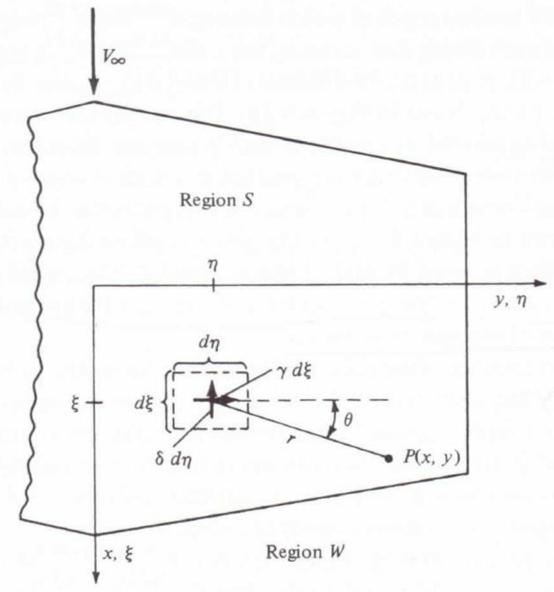
❖ Prandtl's calssical lifting-line theory

➔ Inappropriate for low-aspect-ratio wings and swept wings

➔ A more sophisticated model must be used

❖ Lifting surface theory

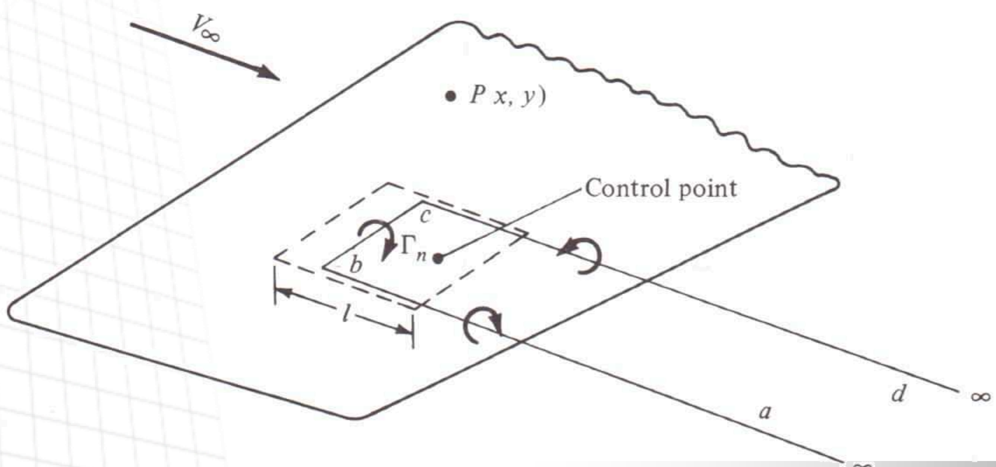
$$|\ dV\ | = \left| \frac{\Gamma}{4\pi} \frac{dl \times x}{|\ r\ |^3} \right| = \frac{\gamma d\xi}{4\pi} \frac{(d\eta) r \sin\theta}{r^3}$$



< 5.5. The Lifting-Surface Theory and VLM Method >

❖ Vortex lattice method

- Bound vortex filament
  - ➔ $l/4$ from the front of the panel
- Control point
  - ➔ Centerline of the panel at a distance $3l/4$ from the front
- Boundary condition
  - ➔ $V_n = V_\infty \sin\alpha + w = 0$



< 6.1 Three-Dimensional Incompressible Flow >

* Irrotational

$$\nabla \times \vec{V} = 0$$

Velocity field → $\vec{V} = \nabla \phi$ ← Velocity potential

$$\nabla \times (\nabla \phi) = 0$$

* Incompressible

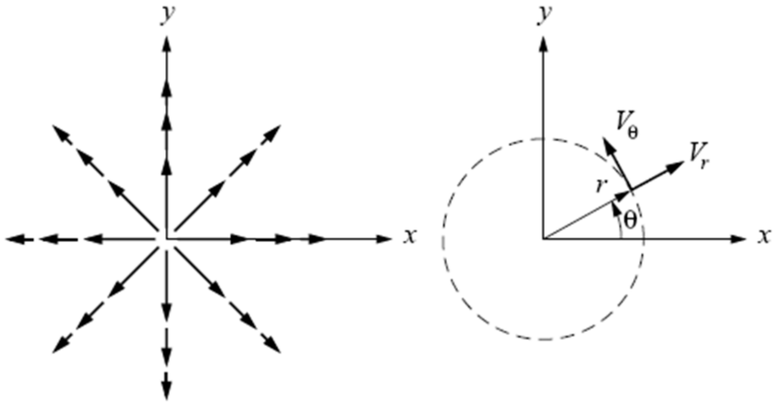
$$\cancel{\frac{\partial \rho}{\partial t}} + \rho(\nabla \cdot \vec{V}) = 0$$: continuity

$$\nabla \cdot \vec{V} = 0$$

$$\nabla \cdot \nabla \phi = \nabla^2 \phi = 0$$

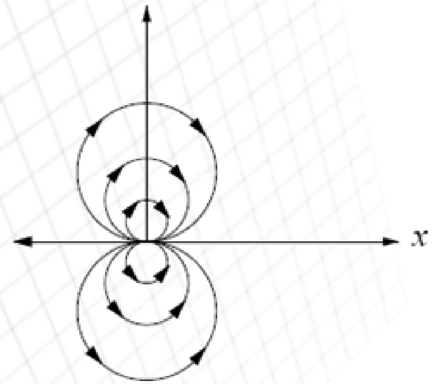
➔ Laplace's Equation

< 6.2 Source >



2D	3D
$S = 2\pi r V_r$	$S = 4\pi r^2 V_r$
$V_r = \dfrac{S}{2\pi r}$	$V_r = \dfrac{S}{4\pi r^2}$
$\phi = \dfrac{S}{2\pi} \ln r$	$\phi = -\dfrac{S}{4\pi r}$
$\psi = \dfrac{S}{2\pi}\theta$	$\psi = \dfrac{S}{4\pi r}\theta$

< 6.3 Doublet >



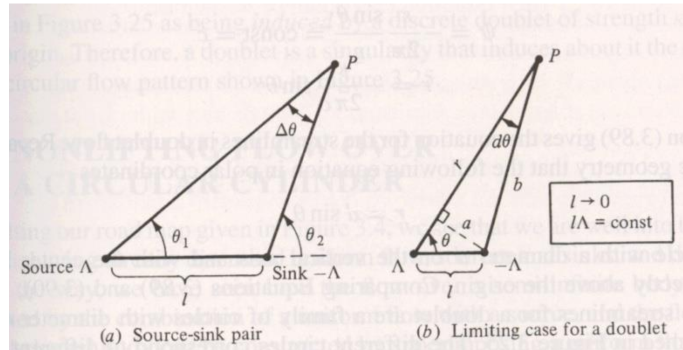


Figure 3.24 How a source-sink pair approaches a doublet in the limiting case.

2D

$$\Delta\theta = \theta_2 - \theta_1 \qquad \psi = \lim_{l\to 0}\left(-\frac{\Lambda}{2\pi}d\theta\right)$$

$$d\theta \simeq \frac{l sin\theta}{r - l cos\theta}$$

$$\psi = \lim_{l\to 0}\left(-\frac{\Lambda}{2\pi}\frac{l sin\theta}{(r - l cos\theta)}\right) = -\frac{\mu \sin\theta}{2\pi r}$$

$$\phi = \frac{\mu}{2\pi}\frac{\cos\theta}{r}$$

3D

$$\phi = -\frac{S}{4\pi}\left(\frac{1}{r_1} - \frac{1}{r}\right) = -\frac{S}{4\pi}\frac{r - r_1}{r_1 r}$$

$$= -\lim_{l\to 0}\frac{S}{4\pi}\frac{r - r_1}{r_1 r} = -\frac{S}{4\pi}\frac{l cos\theta}{r^2}$$

$$= -\frac{\mu}{4\pi}\frac{\cos\theta}{r^2}$$

< 6.4 Flow over a Cylinder / Sphere >

2D

$$\phi = V_\infty r cos\theta + \frac{\mu}{2\pi}\frac{\cos\theta}{r}$$

$$V_r = \frac{\partial \phi}{\partial r} = V_\infty \cos\theta - \frac{\mu}{2\pi}\frac{\cos\theta}{r^2}$$

$$V_{r_{r=a,\theta=\pi}} = - V_\infty + \frac{\mu}{2\pi a^2} = 0$$

$$\therefore \mu = 2\pi a^2 V_\infty$$

$$V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = - V_\infty \sin\theta - a^2 V_\infty \frac{\sin\theta}{r^2}$$

$$\Rightarrow V_{\theta_{r=a}} = -2V_\infty \sin\theta$$

$$\Rightarrow C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - 4\sin^2\theta$$

3D

$$\phi = V_\infty r cos\theta + \frac{\mu \cos\theta}{4\pi r^2}$$

$$V_r = \frac{\partial \phi}{\partial r} = V_\infty \cos\theta - \frac{2\mu \cos\theta}{4\pi r^3}$$

$$V_{r_{r=a,\theta=\pi}} = - V_\infty + \frac{\mu}{2\pi a^3} = 0$$

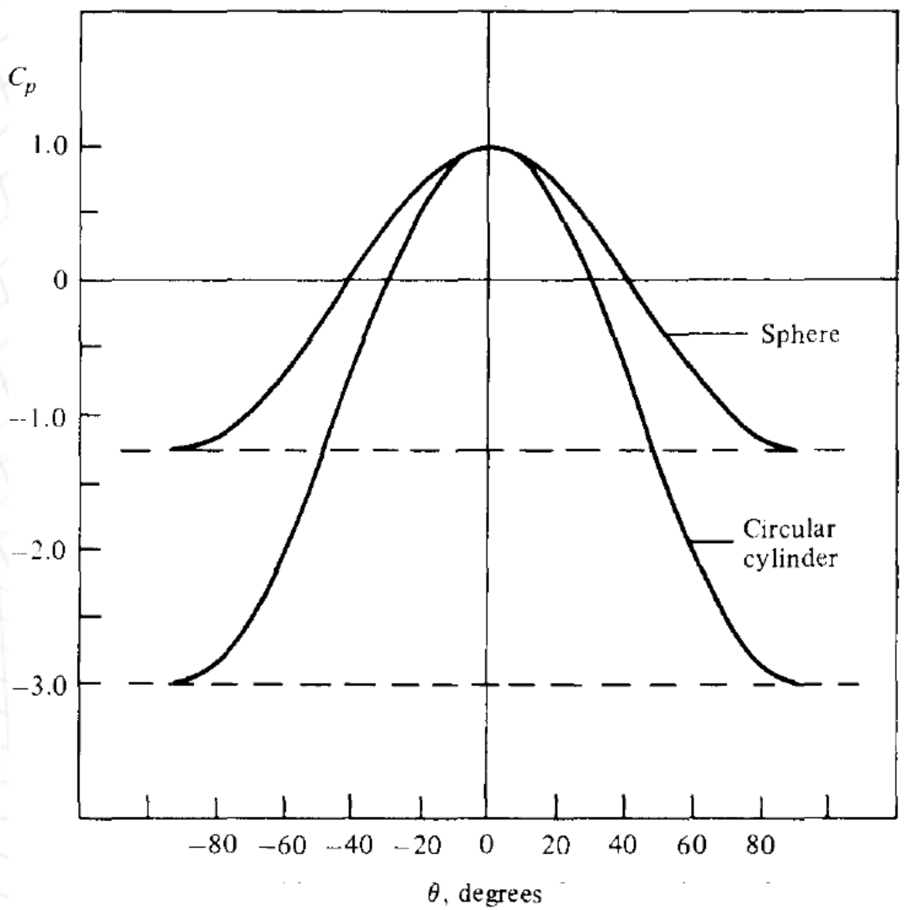
$$\therefore \mu = 2\pi a^3 V_\infty$$

$$V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = - V_\infty \sin\theta - \frac{2\pi a^3 V_\infty \sin\theta}{4\pi r^3}$$

$$\Rightarrow V_{\theta_{r=a}} = - V_\infty \sin\theta - \frac{V_\infty \sin\theta}{2} = -\frac{3V_\infty}{2} sin\theta$$

$$\Rightarrow C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \frac{9}{4} sin^2\theta$$

< 6.4 Flow over a Cylinder / Sphere >



<The pressure distribution over the surface of a sphere and a cylinder>